the normal form of which is obtained by the substitution $s(\zeta) = \zeta^{1/2}q(\zeta)$ with the result

$$\frac{d^2q}{d\zeta^2} + \left[-\frac{3}{4\zeta^2} + \frac{1}{\zeta(\zeta-1)} + \frac{9}{4}\frac{\nu^2\zeta}{\sigma^3}\right]q = 0 \quad (18)$$

It has not been possible to relate Eqs. (17) and (18) to any known equations, but the following observations are somewhat suggestive. For $\nu = 0$, Eq. (17) reduces to the hypergeometric equation, and for very large ζ, Eq. (18) reduces to one that has Airy functions as solutions. Although a large number of transformations has been tried in an attempt to exploit these observations, e.g., solutions as products of known functions, no usable results have been obtained.

In summary, it is felt that the indicated Fourier superposition of the solutions of Eq. (16) provides a numerical algorithm for determining the requisite Riemann function, but Eq. (17) remains a worthy candidate for further research.

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Beam Current Measuring Device for Ion Engine Research

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THE ion beam exiting from an ion rocket usually is measured by metering the electron ground current. Beam current measured in this way usually is correct, but no information concerning the partially neutralized or remaining net current is obtained.

An instrument to measure net direct-current at any position along an ion beam is the magnetic ammeter that detects the magnetic field created by the flow of charged particles. The feasibility of this concept was demonstrated in Ref. 1, where the current of an electron beam passing through a 6-cm-diam mumetal ring was measured. The present study was undertaken to find out if a much larger instrument would have sufficient sensitivity to prove useful in ion engine research.

As shown in Fig. 1, the ammeter consists of an indium arsenide probe mounted in the air gap of a high permeability ring. The voltage signal from the Hall effect transducer is detected with a commercial gaussmeter.

The current of ions passing through the ring can be considered to be a one-turn coil, and the magnetic field in the air gap is calculated from the magnetic circuit equation. For a small gap, the magnetic flux Φ is constant around the ring, and[4]

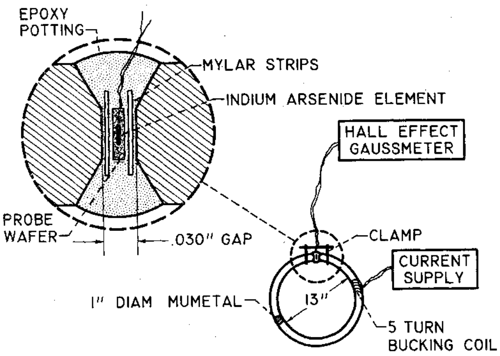

Fig. 1 Mumetal ring assembly

$$\Phi \oint \frac{dl}{\mu A} = I \quad (1)$$

which can be written as

$$\Phi[(l_0/\mu_0A_0) + (l_m/\mu_mA_m)] = I \quad (2)$$

where l_0, A_0, and μ_0 are the length, area, and permeability of the air gap, and l_m, A_m, and μ_m refer to the same quantities in the metal ring.

The sensitivity of the device can be defined as the ratio of the magnetic field in the air gap to the current passing through the ring:

$$S = B_0/I \quad (3)$$

where $B_0 = \Phi/A_0$. From (2)

$$S = \frac{1}{(l_0/\mu_0) + (l_mA_0/\mu_mA_m)} \quad (4)$$

In the present case $(l_0/\mu_0) >> (l_mA_0/\mu_mA_m)$, so that the sensitivity depends only on the characteristics of the air gap. The calculated sensitivity of the present device $S = \mu_0/l_0 =$ 0.02 gauss/ma is in agreement with the measured value. The ring can be made indefinitely large and still have the sensitivity dependent only on air gap characteristics as long as

$$(l_m/l_0)(\mu_0/\mu_m)(A_0/A_m) << 1 \quad (5)$$

For an area ratio A_0/A_m of unit order, the length ratio l_m/l_0 should be much less than the permeability ratio μ_0/μ_m. If the inequality (5) is not satisfied, the device will require a more careful calibration.

In practice the magnetic field is not measured directly. Current I_c through the N turn bucking coil is used to cancel the field in the ring. At the null condition the beam current is NI_c or $5I_c$ for the five-turn bucking coil used.

The greatest inaccuracy in the device is d.c. drift caused by the extreme sensitivity of the probe to temperature. When a

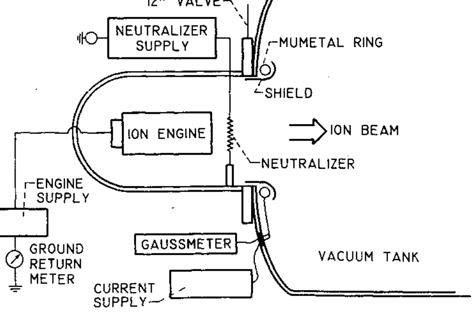

Fig. 2 Test setup

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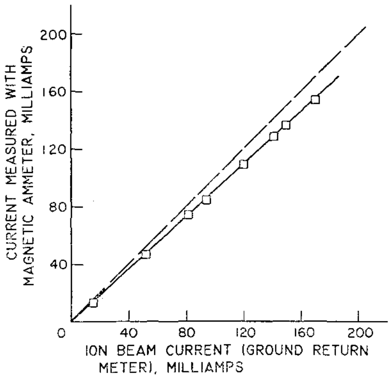

Fig. 3 Measurement of beam current without neutralizer

source of heat is present, either radiative or from ion impingement, cooling is required.

Bench tests of the ammeter with a current carrying wire simulating the ion beam show that the instrument reads correctly within ordinary meter accuracy (3%). Movement of the current-carrying wire within the ring did not affect the readings. The device also is sensitive to any stray magnetic fields and must be nulled before a run.

Application to Ion Engine Studies

Figure 2 shows the ammeter set up inside a vacuum tank for preliminary tests with an electron bombardment-type ion engine. Details of the engine and vacuum facility can be found in Refs. 2 and 3.

In order to compare the magnetic ammeter with a ground return meter, the ion engine was operated without current neutralization. The result is shown in Fig. 3. The current through the ring is about 9% lower due to a loss of ions from charge exchange and excessive beam spreading. It should be noted that electrons coming upstream through the ring are reflected at the engine and are not counted as current since they return through the ring again.

As shown in Fig. 4, the ammeter was used to measure the degree of beam current neutralization with an immersion-type neutralizer in operation. The neutralizer was a 0.020-in. tungsten wire stretched across the beam 3 in. from the engine. Ideally, one might expect zero current through the ring when the neutralizer current is equal to the beam current (space application). However, there is an electron loss current to the valve and tank walls due to the thermal motion of the electrons.

It is concluded that the magnetic ammeter is a practical device for measuring net beam current and has the important advantage that no physical object is placed in the beam, so that the beam itself is not affected.

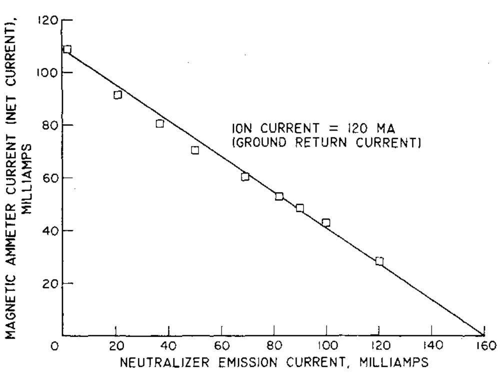

Fig. 4 Current through magnetic ammeter as a function of neutralizer emission current for constant ion current of 120 ma

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Effect of Radiation on Flow Properties behind Strong Shock Waves in Air

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Many of the problems associated with superorbital re-entry can be studied in high-performance shock tubes. Shock Mach numbers between 25 and 40 are required to simulate the stagnation enthalpies encountered by vehicles returning to earth on hyperbolic trajectories. (The equivalent flight velocity, in thousands of feet per second, is approximately 1.5 times the incident shock Mach number.) The temperature behind the shock at these Mach numbers is so high that thermal radiation has a significant effect on the flow properties behind the shock.

A low-pressure, arc-driven shock tube with a 12-in.-i.d. driven section currently is under construction at Lockheed Missiles and Space Company. In this facility, it will be possible to study equilibrium and nonequilibrium radiation as well as convective heating at total enthalpies corresponding to superorbital re-entry. In this note, the results of a preliminary investigation of the effects of thermal radiation on driven-gas properties are presented for the case of air. The initial conditions and Mach numbers selected in the numerical examples are typical of those expected in the LMSC low-pressure, arc-driven shock tube.

It can be shown that the effect of energy loss due to radiation is to reduce the speed of the shock wave and increase the speed of the contact surface. Conditions behind the shock wave therefore will change throughout its passage from the diaphragm station to the working section. However, a study of the effects of radiation cooling on shock waves traveling with constant speed provides valuable information about the ranges of shock Mach number and initial pressure over which the radiation loss will have an appreciable effect on shock-tube performance.

The equations governing the flow through a constant-speed shock wave are, in a coordinate system fixed in the wave,

$$\rho_2 v_2 = \rho_1 v_1 \tag{1}$$

$$p_2 + \rho_2 v_2^2 = p_1 + \rho_1 v_1^2 \tag{2}$$

$$h_2 + \tfrac{1}{2} v_2^2 = h_1 + \tfrac{1}{2} v_1^2 - q \tag{3}$$

where ρ is density, v velocity, p pressure, and h enthalpy. Subscript 1 refers to conditions ahead of the wave and subscript 2 to any point behind the wave; q is the total amount of energy lost per unit mass up to station 2. For $q = 0$, the

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